Model Identification and Comparative Study of DC Motor Speed Control for Set Point Tracking and Disturbance Rejection

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Abstract— *Modelling and Identification of dynamic systems constitute an essential stage in practical control and applications. This paper focuses on model identification of an armature controlled laboratory DC motor, rotating in one direction and major techniques for tracking set point commands and suppressing sensitivity to load disturbances. Input-output data of the motor is acquired offline, with the aid of a MATLAB implemented recursive least square algorithm, a linear model transfer function having two poles-one zero is obtained, subsequently simulations in normal MATLAB program and Simulink are carried out to assess the model transient and steady state performances. Result of simulation show that the effect of second control loop secure the integral feedback control point and also secure the target speed. Linear Quadratic Regulator (LQR) gives stable and steady speed on close loop with respect to load and shows that the LQR compensator performs best at rejecting load disturbances.*

Keywords— *DC motor modelling,* Load Torque Condition*, LQR compensator, System identification, Tracking set point*

1. INTRODUCTION

The applications of direct current (DC) Motors as components of electromechanical systems have remained vital as actuating elements in industrial processes for their advantages of easy speed and position control and wide adjustability range (Sandeep, 2016). DC Motors have been widely used in many industrial applications such as electric vehicles, steel rolling mills, electric cranes, robotic manipulators etc., due to its precise, wide, simple, and continuous control characteristics. Effective use of dc motors requires sound knowledge of the system components and their dynamic characteristics (Shuang, 2010).

DC motors are supposed to operate with high accuracy and speed despite adverse effects of system nonlinearities and uncertainties. This robustness property is of great importance if the system is part of a robotic or servo system, which requires insensitivity to unmoulded dynamics (Lischinsky,1999). Effect of small load on the motor is absorbed by the kinetic energy of the rotor, under large load change, limitation is imposed on the rotor due to nonlinear behaviour of the magnetization characteristic of the winding circuits (Aisha *et al.*, 2015). The overall effect of such disturbance is that it minimises the speed of the motor and reduces its robustness for specific applications. Several works on Modelling and control of DC motor for performance improvement has been reported in some literatures.

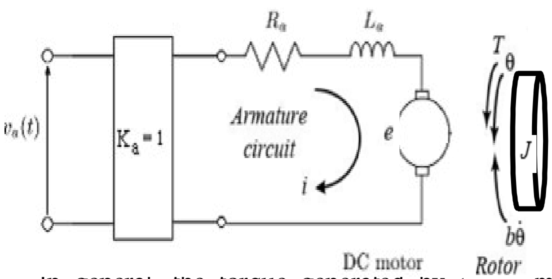
A Simulation of a speed control of DC motor drive using Genetic algorithm (GA) turned PID controller in MATLAB was presented in Shrabani et al., (2013). The work used GA for tuning the speed controller parameter, and a comparative analysis of proposed technique with the classical method of PID control system applied to DC motor drive was presented. The study shows that the proposed controller enhances the performance of speed control of DC motor drive than the conventional PID controller.

Jide *et al.*, 2015 2015 proposed the feedback method for armature controlled DC motor speed control to enhance the performance of the transient response of the DC motor. The aim was to establish relationship between the speed of DC motor and the load torque at different voltages as well as investigating the performance of closed-loop systems when different voltages are applied to the armature circuit of the motor when a constant voltage is supplied to the field circuit of the motor. The proposed feedback method performance analysis of the modelled system shows that usage of feedback approach enhances the performance of the transient response of the armature controlled DC motor. Someshwar *et al.*, (2016) work proposed a comparative analysis of the DC motor using LAG compensator and PID controller for optimizing the performance for a DC motor. In the work, Root Locus plot, Bode plot and step response were employed to analysed the system and determine its performance improvement. In the work, a LAG compensator and PID controller was applied on an unstable DC motor model in order to obtain a stable model of DC motor for optimal performance.

Most of the cited work adopted models that are analytically derived but in this work, a system identification (SI) technique using offline data acquisition is proposed and analysis of the system will be carried out for transient and steady state performance. In addition to this, the paper will design and compare three DC motor control techniques namely feedforward command, integral feedback ward control and LQR control for tracking set point and reducing sensitivity to load disturbances to a laboratory DC Motor model provided on TPS-3011. The rest of the work is organised as follows: Section 2 presents the review of a dc motor. Section 3 is dedicated to model identification algorithm analysis and development. Section 4 presents the model performance while section 5 presents the dc motor control design and implementation. Finally, the conclusion of the work is summarized in section 6.

2. REVIEW OF MODEL DEFINITION

An equivalent circuit model of the motor is shown in Figure1



In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. Assuming that the magnetic field is constant, therefore, the motor torque (T) is proportional to the armature current i by a constant factor k_t as shown in equation (1). This is referred to as an armature-controlled motor.

$$T = K_t i \quad (1)$$

Mathematically, the back *e.m.f*, e, is defined as follow:

$$e = K_e \omega \quad (2)$$

where ω is the angular velocity of the shaft and K_e is a constant factor of proportionality. In SI units, the motor torque and back *e.m.f* constants are equal, that is, $K_t = K_e$ therefore, we will use K to represent both the motor torque constant and the back *e.m.f* constant. From Figure 1, the following governing equations are derive based on Newton's 2nd law and Kirchhoff's voltage law.

$$J\dot{\omega} + b\omega = Ki \quad (3)$$

$$L\frac{di}{dt} + Ri = V_a - K\omega \quad (4)$$

Since the intended model structure for the system in this study is the transfer function model, applying the Laplace transform to equations (3) and (4) in terms of the Laplace variable *s* yield equations (5) and (6)

$$s(Js + b)\theta(s) = Ki(s) \quad (5)$$

$$(Ls + R)I(s) = V(s) - K(s)\theta(s) \quad (6)$$

The open-loop transfer function after eliminating $I(s)$ between equations (5) and (6) and relating the rotational speed and the armature voltage yields:

$$G_v(s) = \frac{\omega(s)}{[illegible]} = \frac{K}{[illegible]} \quad (7)$$

where $\omega(s)$ is rotor speed, $V(s)$ is input voltage, R armature resistance, J is rotor inertia, b is viscous friction constant, and K is back *e.m.f.* constant.

2.1 Model Identification

If input u, and output v is put in vector form as:

$$P = [u(1), v(1), \ldots, u(N), v(N)]^T$$

Then a compact model can be written as:

$$y(t) = \varphi^T(t)\theta \quad (8)$$

where;

$$\theta = [a1, \ldots cn, b1, \ldots, bm]T \text{ and}$$

$$\varphi = [-y(t-1), \ldots, -y(t-n)\ u(t-1)\ \ldots, u(t-m)]^T$$

are vectors of estimated parameters and past values of input-output vector respectively. A recursive least square for estimating the parameter vector θ is given as Ljung (1987):

$$\hat{\theta}^N - [\sum_{t=1}^{N} \varphi^T(t)\varphi(t)]^{-1} \sum_{t=1}^{N} \varphi(t)y(t) \quad (9)$$

However, the MATLAB identification toolbox was used to generate a set of codes that generate transfer function from the estimated parameter employ in equation (9).

In modelling a DC motor connected to a load via a shaft, the general approach is to neglect the nonlinear effects and build a linear transfer function representation for the input–output relationship of the DC motor and the load it drives (Nayana *et al.*, 2013). This assumption is satisfactorily accurate and valid as far as conventional control problems are concerned. Giving a system input-output data, a numerical software such as MATLAB implements recursively the algorithm and gives an estimate of the parametric model of the system in form of state space, transfer function or ARX model.

2.1.1 System Data Acquisition

The input and output signals are sampled offline at an estimated time interval of 5 seconds. Input is applied through fixed amplifier gain K from the on-board potentiometer. Output is a speed tacho-generator providing dc voltage as a measure of the motor speed. The test bed for collection of data is shown in Figure2.

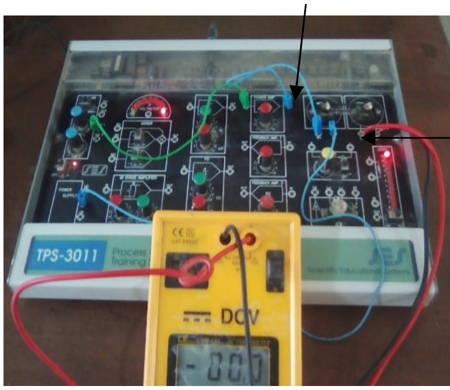


Figure 2: Process Kit with Offline DC Motor Data Acquisition Set Up

The number of input-output sampled obtained was 160 and the data samples is presented in Appendix A.

2.1.2 Model Estimation

Accurate model building is a crucial stage in practical control problems. An adequately developed system model is essential for reliability of the designed control. When the plant has uncertainties or time dependencies, or cannot be parameterized, a model for the system may be hard to obtain. For such systems, the system parameters should be determined using system identification techniques. In light of this, MATLAB identification toolbox code for model estimation is provided with the data after pre-processing. Amplifier gain is fixed at $K_p = 0.9791$ representing the dc gain of the motor. Using the Predictive Error algorithm (PEM) with specified structure having two poles, no zero, MATLAB returns the following model structure:

$$G(s) = \frac{K_p(s\tau_z+1)}{(s\tau_1+1)(s\tau_2+1)} \tag{10}$$

The poles time constants estimates are: τ_1 = 0.071, τ_2 = 0.071. And zero-time constant estimate is τ_z = 1.94. With the given time constants estimated and substituted into (10), the systems transfer function is given as:

$$G(s) = \frac{0.981(1.943s+1)}{[illegible]} \tag{11}$$

3. SYSTEM PERFORMANCE ANALYSIS AND SIMULATION

Performance of industrial systems are greatly affected by the following types of nonlinearities:

1. Parameter variations
2. Sudden shock; due to large surge in their primary signal input or at some location along the signal path
3. Signal Impact that is sustained
4. Noise in measurement
5. Load variations
6. Nonlinearities in many parts of the system

In this work, the model was simulated based on the following two case scenarios: Case1 is based on Model response under no load condition while Case 2 is Model response under load torque condition with compensation.

3.1 Model response under no load condition,

The response of the developed model to a unit step input case is obtained and step response plot presented in Figure 3

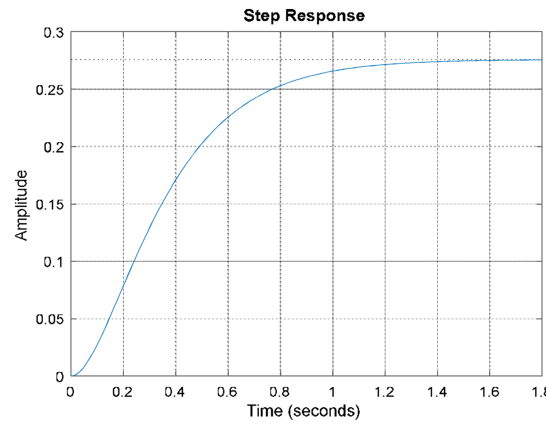


Figure 3: Model response to unit step

Figure 3 shows that the system has large steady state error of about 74.5% while the settling time was about 1.6 seconds.

3.2 Model Response Under Load Torque Condition with Compensation

In this case, the systems response was obtained using different control techniques while subjecting the system to disturbance perturbation. The implementation and results are described as follow:

3.2.1 Feedforward DC Motor Control Design

The feed forward control structure to command the angular velocity w to a given value $\omega_{ref}$is presented in Figure 4

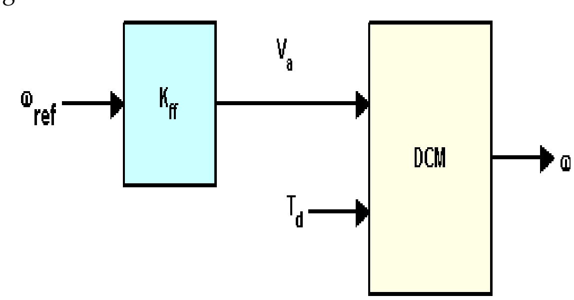


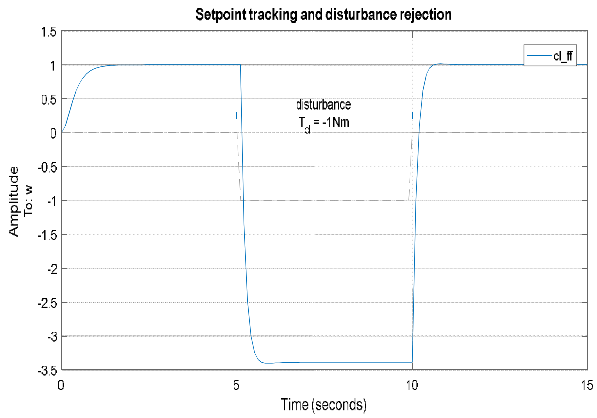
Figure 4: Feedforward control structure

The Feedforward is an open loop system with Voltage control V_a. Taking K_{ff} as DC motor gain, which is defined and set to the reciprocal of the DC gain from V_a to ω.

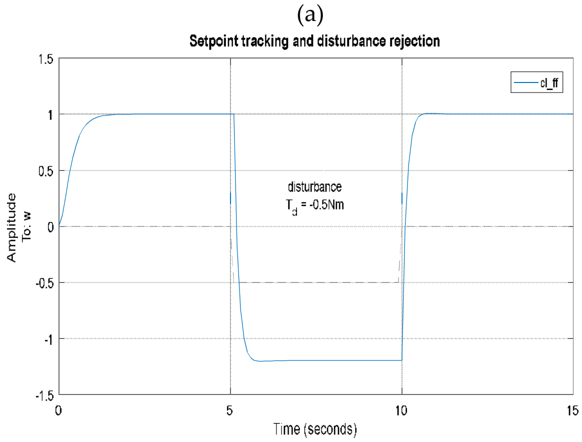
$$K_{ff} = \frac{1}{[illegible]} * (dcm(1)) \tag{12}$$

where; $K_{ff} = 3.5049$

To evaluate the feedforward design in the face of load disturbances, the systems response to a step command $\omega_{ref} = 1$ with a disturbance $T_d = -0.5\ Nm$ and $-1.0\ Nm$ between $t = 5s$ and $t = 10s$ were simulated and results are presented in Figure 5 (a) and (b) respectively.



(a)



(b)

Figure 5 *(a) and (b):* DC motor Stepping and disturbance rejection for feedforward control

3.2.2 Feedback DC Motor Control Design (Root locus)

To enforce zero steady-state error, the use of integral control as shown in equation (12)

$$C(s) = {}^{K}/_{c} \qquad (13)$$

To determine the gain K, the root locus technique is applied to the open-loop $\frac{1}{s} * \mathrm{Transfer}[V_a -> \omega]$. Figure 6 shows the complete control structure for this approach.

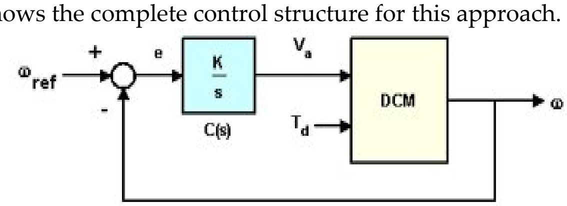


Figure 6: Feed backward control structure

The main idea of root locus design is to obtain the closed-loop response from the open-loop root locus plot for the DC motor response. By adding zeroes and poles to the original system, the root locus is modifiable using a new closed-loop response. Foremost, the root-locus for the system itself imposed with a unit circle. Figure 7 shows the root locus plot used for determining the feed backward closed-loop gain.

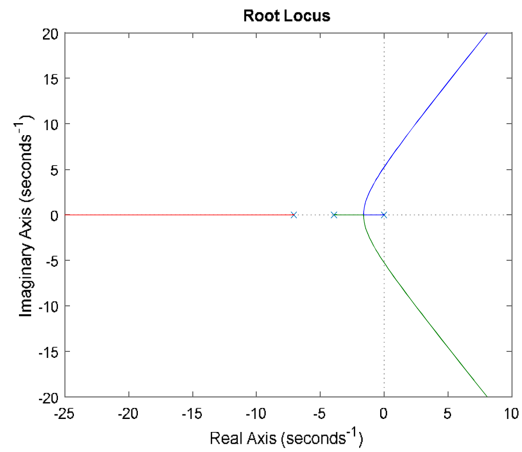
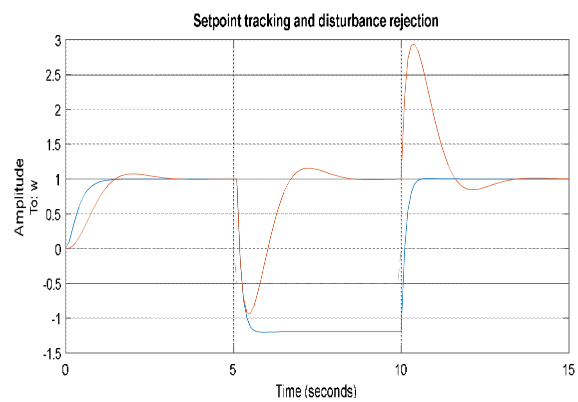
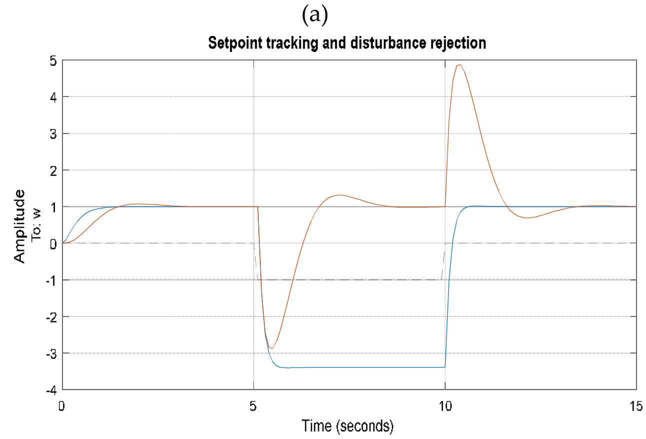


Figure 7: Root Locus design method for the DC motor system

The gain value is read from the plot to be 4.5, but a reasonable choice used in this work is K = 5. By Comparing this new design with the initial feed forward design on the same test case, plots for the stepping and disturbance rejection are presented in Figure 8(a) and (b):



(a)



(b)

Figure 8(a) and (b): DC motor Stepping and disturbance rejection for feedforward and feed backward control

3.2.3 LQR DC Motor Control Design

To further improve performance, try designing a linear quadratic regulator (LQR) for the feedback structure shown Figure 9:

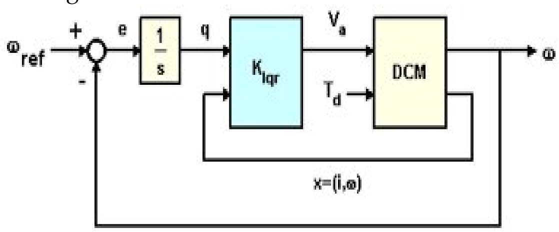
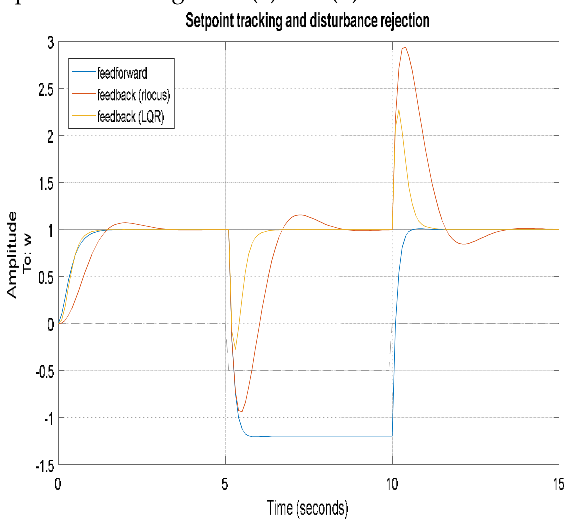
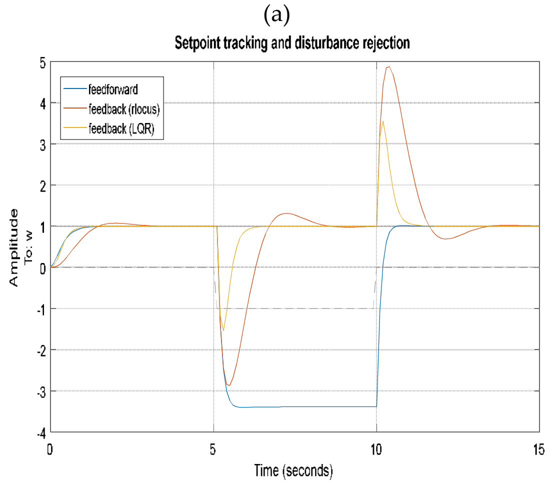


Figure.9: LQR control structure

In addition to the integral of error, the LQR scheme also uses the state vector $x = (i, w)$ to synthesize the driving voltage Va. The optimal LQR gain for this cost function is computed and the closed-loop model response is presented in Figure 10(a) and (b).



(a)



(b)

Figure 10 (a) and (b): DC motor Stepping and disturbance rejection for the 3 control techniques

4. DISCUSSION OF RESULTS

Feedback (closed-loop) control can be used to stabilize systems, speed up the transient response of the DC motor, improve the steady-state characteristics, provide disturbance rejection state, and decrease the sensitivity to parameter variations. From the result shown in figure 5(a) and (b), it can be observed clearly that the feed forward control handles load disturbances poorly as it remained sensitive to the disturbance over the entire range of time.

The root locus design approach proves to be better at rejecting load disturbances as compared to the feed forward approach and this can be seen in Figure 9(a) and Figure 9 (b) for load torque of -0.5 and -1 Nm respectively. Figure 10(a) and Figure 10(b) is a plot of the system response to the three (3) control techniques and the LQR compensator performs best at rejecting load disturbances (among the three DC motor control designs discussed). Table 1, shows the summary of the system response for both steady state and transient state performance for the 3 control technique

Table 1: Performance indices for load torque of -0.5 and -1.0 Nm

Technique	Rise time	Settling time with load torque	Steady state error
Feed forward	1.80	6.00	0.00
Feed backward	2.00	3.20	0.00
LQR control	1.80	1.20	0.00

5. CONCLUSION

The result shows that the developed is in conformity with other work. DC Motor model identification and control designed has been successfully achieved using MATLAB program to track the motor speed set point under loading and without load. The simulation results show that in terms of steady state, both techniques perform robustly while in terms of transient performance characterized by disturbance rejection capabilities, the LQR technique has a better performance amongst the other three control techniques.

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Appendix A: Dc Motor Input-Output Data

S/N	INPUT (V)	OUTPUT (V)	S/N	INPUT (V)	OUTPUT (V)	S/N	INPUT (V)	OUTPUT (V)	S/N	INPUT (V)	OUTPUT (V)
1	0.96	0.21	41	4.88	8.53	81	2.34	2.93	121	4.02	6.16
2	1.09	0.49	42	4.86	8.22	82	2.27	2.81	122	4.05	6.32
3	1.25	0.77	43	4.82	7.99	83	2.14	2.48	123	4.12	6.42
4	1.55	1.35	44	4.80	7.87	84	2.02	2.32	124	4.22	6.58
5	1.66	1.52	45	4.78	7.72	85	1.94	2.29	125	4.28	6.68
6	1.74	1.68	46	4.67	7.51	86	1.91	2.18	126	4.31	6.81
7	1.87	1.93	47	4.55	7.25	87	1.89	2.03	127	4.37	6.95
8	1.97	2.14	48	4.46	7.07	88	1.67	1.74	128	4.49	7.11
9	2.06	2.30	49	4.36	6.88	89	1.65	1.63	129	4.51	7.22
10	2.14	2.46	50	4.21	6.58	90	1.56	1.48	130	4.54	7.27
11	2.31	2.80	51	4.16	6.46	91	1.54	1.41	131	4.57	7.35
12	3.46	5.12	52	4.08	6.31	92	1.44	1.36	132	4.66	7.43
13	3.58	5.38	53	4.01	6.18	93	1.42	1.30	133	4.73	7.62
14	3.67	5.52	54	3.97	6.10	94	1.38	1.25	134	4.80	7.79
15	3.73	5.66	55	3.93	6.01	95	1.21	1.20	135	4.82	7.83
16	3.77	5.76	56	3.87	5.95	96	1.18	1.15	136	4.88	7.89
17	3.82	5.88	57	3.82	5.87	97	1.08	1.09	137	4.91	7.98
18	3.84	5.93	58	3.77	5.72	98	1.05	1.01	138	4.96	8.29
19	3.90	6.01	59	3.67	5.46	99	1.03	0.73	139	4.94	8.57
20	3.96	6.09	60	3.61	5.38	100	0.69	0.48	140	4.90	8.02
21	4.00	6.16	61	3.51	5.19	101	0.86	0.21	141	4.88	7.99
22	4.07	6.32	62	3.42	5.01	102	1.08	0.49	142	4.87	7.87
23	4.12	6.42	63	3.32	4.82	103	1.22	0.77	143	4.82	7.72
24	4.20	6.58	64	3.27	4.72	104	1.53	1.35	144	4.79	7.49
25	4.26	6.68	65	3.21	4.62	105	1.60	1.52	145	4.76	7.35
26	4.32	6.81	66	3.16	4.49	106	1.77	1.68	146	4.65	7.08
27	4.39	6.95	67	3.09	4.37	107	1.87	1.93	147	4.54	6.74
28	4.47	7.11	68	2.99	4.18	108	1.93	2.14	148	4.46	6.48
29	4.53	7.22	69	2.91	4.02	109	2.00	2.30	149	4.38	6.36
30	4.55	7.27	70	2.83	3.87	110	2.18	2.46	150	4.27	6.29
31	4.59	7.35	71	2.75	3.71	111	2.51	2.80	151	4.20	6.18
32	4.63	7.43	72	2.69	3.59	112	3.47	5.12	152	4.16	6.10
33	4.73	7.62	73	2.61	3.43	113	3.59	5.38	153	4.09	6.01
34	4.81	7.79	74	2.53	3.26	114	3.66	5.52	154	3.99	5.95
35	4.84	7.83	75	2.43	3.07	115	3.75	5.66	155	3.93	5.87
36	4.87	7.89	76	2.32	2.87	116	3.77	5.76	156	3.88	5.72
37	4.91	7.98	77	2.18	2.64	117	3.80	5.88	157	3.80	5.46
38	4.97	8.09	78	2.14	2.59	118	3.82	5.93	158	3.77	5.38
39	4.94	8.12	79	2.09	2.51	119	3.89	6.01	159	3.67	5.19
40	4.90	8.18	80	2.04	2.43	120	3.96	6.09	160	3.52	5.01